

SET-3प्रश्न-पत्र कोड
Q.P. Code**65/3/3**

गणित MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

- Please check that this question paper contains **23** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. []



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1 to 18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21 to 25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26 to 31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32 to 35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36 to 38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then :

(A) $ \vec{a} = \sqrt{2}$	(B) $ \vec{b} = \sqrt{2}$
(C) $ \vec{b} = 10\sqrt{2}$	(D) $ \vec{a} = \frac{10}{\sqrt{2}}$
2. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then :

(A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$	(B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
(C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$	(D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 $
3. Direction ratios of lines l_1 and l_2 respectively are $\langle 1, 0, 0 \rangle$ and $\langle 0, -1, 0 \rangle$. The direction ratios of the line perpendicular to both l_1 and l_2 are :

(A) $\langle 1, 1, 0 \rangle$	(B) $\langle 0, 0, -1 \rangle$
(C) $\langle 1, 1, 1 \rangle$	(D) $\langle 1, 0, -1 \rangle$



4. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E | F) - P(F | E)$ is equal to :
- (A) $\frac{2}{7}$ (B) $\frac{3}{35}$
(C) $\frac{1}{70}$ (D) $\frac{1}{7}$
5. The domain of $\cos^{-1}(4x + 1)$ is :
- (A) $[-1, 1]$ (B) $[-3, 5]$
(C) $[-4, 4]$ (D) $[-\frac{1}{2}, 0]$
6. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :
- (A) -1 (B) 1
(C) $\frac{5}{3}$ (D) 7
7. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an :
- (A) symmetric matrix
(B) skew-symmetric matrix
(C) null matrix
(D) identity matrix
8. If a matrix X is such that $[2 \ 1] X = [3 \ 4 \ 5]$, then the order of matrix X is :
- (A) 3×1 (B) 2×3
(C) 1×2 (D) 1×3
9. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be :
- (A) 7 (B) 5
(C) -7 (D) -1
10. If $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|2A|$ is :
- (A) 6 (B) 54
(C) 12 (D) 24



11. The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is :
- (A) $\frac{1}{4}$ (B) 2
(C) $\frac{1}{2}$ (D) 0
12. If $2 \cos^{-1} x = y$, then $\frac{dy}{dx}$ is :
- (A) $-2 \sin^{-1} x$ (B) $-\frac{1}{2} \sin \frac{y}{2}$
(C) $\frac{-1}{\sqrt{1-2x^2}}$ (D) $-2 \operatorname{cosec} \frac{y}{2}$
13. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is :
- (A) $400\pi \text{ cm}^3/\text{cm}$ (B) $100\pi \text{ cm}^3/\text{cm}$
(C) $50\pi \text{ cm}^3/\text{cm}$ (D) $25\pi \text{ cm}^3/\text{cm}$
14. $\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx$ is equal to :
- (A) $\tan^{-1}(\sin x) + C$
(B) $\sin^{-1}(\sin x) + C$
(C) $\log |\sin x + \sqrt{1 + \sin^2 x}| + C$
(D) $\log |\sin x - \sqrt{\sin^2 x + 1}| + C$
15. $\int_{-1}^1 (1 - |x|) dx$ is equal to :



(A) $2 \int_0^1 (1 + x) \, dx$

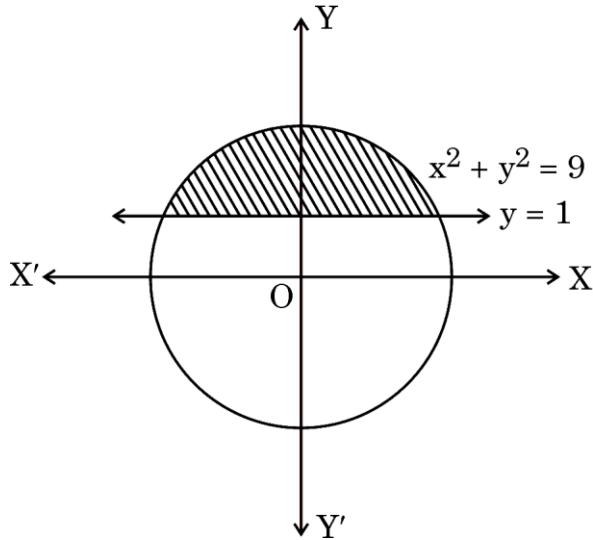
(B) $2 \int_{-1}^0 (1 + x) \, dx$

(C) 0

(D) $2 \int_{-1}^0 (1 - x) \, dx$



16. The area of the shaded region of the circle given below is equal to :



- (A) $\int_1^3 \sqrt{9-y^2} \, dy$ (B) $2 \int_1^3 \sqrt{9-y^2} \, dy$
 (C) $\int_0^3 \sqrt{9-x^2} \, dx$ (D) $2 \int_0^3 \sqrt{9-x^2} \, dx$

17. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ?

- (i) $F(x, y) = 3x + 2y$ (ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$
 (iii) $F(x, y) = e^{y/x} + 1$ (iv) $F(x, y) = \sqrt{x^2 + y^2} - y$
 (A) (i) and (ii) (B) (i), (ii) and (iii)



(C) (ii), (iii) and (iv)

(D) (ii) and (iii)

18. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(A) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$

(B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$

(D) $|\vec{a} \times \vec{b}| \geq |\vec{a}| |\vec{b}|$

Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R) : $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

20. Assertion (A) : The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors.

Reason (R) : $\vec{a} \cdot (-2\vec{a}) = 0$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p.



OR

- (b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.

22. If $x = e^{\sin^{-1} t}$ and $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

- 23.** Find the sub-interval of $(0, \infty)$ in which $f(x) = x \log x$ is increasing.



24. (a) Find the value of $\sin [\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))]$.

OR

- (b) A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1) (3, 3), (1, 2)\}$. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.
25. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A survey was conducted on the patients who have undergone knee replacement surgeries.



It was found that, Robotic Knee replacement surgeries have 90% success rate.

On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :

- (i) exactly one surgery is successful,



- (ii) at most two surgeries are successful.

- 27.** (a) Find :

$$\int \frac{dx}{x^{1/2} + x^{1/3}}$$

OR

- (b) Find :

$$\int \tan^{-1} \left(\frac{1-x}{1+x} \right) dx$$



28. Evaluate :

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx$$

29. (a) Find :

$$\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$$

OR

(b) Find :

$$\int \frac{x + 3}{x^2 + 4x + 5} dx$$

30. (a) Find the general solution of the differential equation

$$(x^2 - y^2) dx + 2xy dy = 0.$$

OR

(b) Solve the differential equation

$$\sin x \cos y dx + \cos x \sin y dy = 0, \text{ given that } y = \frac{\pi}{4} \text{ when } x = 0.$$

31. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

SECTION D



This section comprises 4 Long Answer (LA) type questions of 5 marks each.

- 32.** (a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$$l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

$$l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

OR

- (b) Opposite sides of a square are along the lines :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p .

33. Show that a function $f : R_+ \rightarrow A \subset N$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $R_+ = [0, \infty)$. Also, find if there exists $a \in R_+$ such that $f(a) = 7$. Justify.

34. Solve the following Linear Programming Problem graphically :

Maximize $Z = 8x + 8y$

subject to the constraints

$$x - 20 \leq 0$$

$$2x + 3y \leq 120$$

$$2x + y \leq 60$$

$$x \geq 0, y \geq 0$$

35. (a) On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.



A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

OR

9

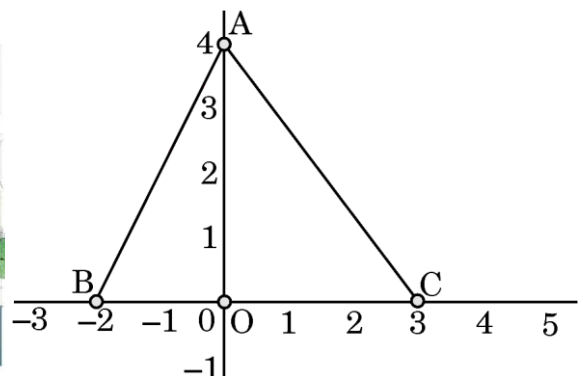
- (b) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. There is a triangular park in the society. The park is divided into two sections as shown in the figure.





In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed.

The vertices of the triangular park ABC are A(0, 4), B(− 2, 0) and C(3, 0).

Based on the above information, answer the following questions :

- (i) Write the equation of the boundary line AB of the park. 1
- (ii) Write the equation of the boundary line AC of the park. 1
- (iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football. 2

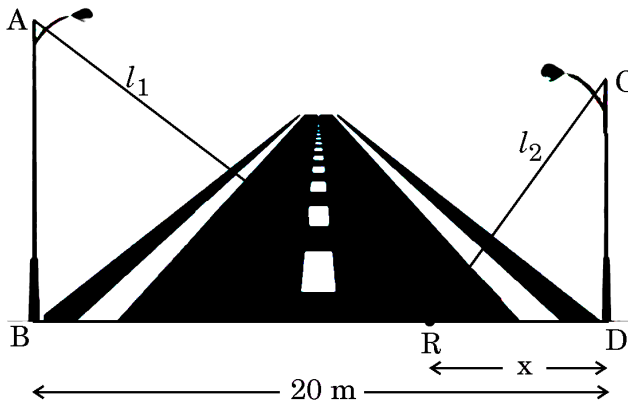
OR

- (iii) (b) Using integration, find the area of region AOB. 2



Case Study – 2

37. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

- (i) Express $p(x) = l_1 + l_2$ in terms of x. 1
- (ii) Find $p'(x)$. 1
- (iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum. 2

OR

- (iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ? 2



Case Study – 3

38. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.



As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.

Based on the above information, answer the following questions.

- (i) Find the probability that a student selected at random is a regular student. 1
- (ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ? 1
- (iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout. 2

OR

- (iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student. 2