

Series : ZXW5Y



SET~3

प्रश्न-पत्र कोड
Q.P. Code

65/5/3

रोल नं.

Roll No.



परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



गणित

MATHEMATICS



निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट

- (I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।
- (II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- (III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।
- (IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- (V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

NOTE

- (I) Please check that this question paper contains **23** printed pages.
- (II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (III) Please check that this question paper contains **38** questions.
- (IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
- (V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is :

(A) $-\frac{\pi}{3}$

(B) $-\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

2. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0

(B) -10

(C) 10

(D) 1

3. If $A = kB$, where A and B are two square matrices of order n and k is a scalar, then :

(A) $|A| = k|B|$

(B) $|A| = k^n|B|$

(C) $|A| = k + |B|$

(D) $|A| = |B|^k$



4. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

is continuous at $x = 0$, then the value of a is :

- (A) 1 (B) -1
(C) ± 1 (D) 0

5. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

- (A) 3 (B) 7
(C) ± 7 (D) ± 3

6. If $P(A \cup B) = 0.9$ and $P(A \cap B) = 0.4$, then $P(\bar{A}) + P(\bar{B})$ is :

- (A) 0.3 (B) 1
(C) 1.3 (D) 0.7

7. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

- (A) $3 \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$
(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

8. Let A and B be two matrices of suitable orders. Then, which of the following is **not** correct ?

- (A) $(A')' = A$ (B) $(kA)' = kA'$, k is a scalar
(C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$

9. The area of the region enclosed between the curve $y = x|x|$, x -axis, $x = -2$ and $x = 2$ is :

- (A) $\frac{8}{3}$ (B) $\frac{16}{3}$
(C) 0 (D) 8



10. If $f(x) = \{[x], x \in \mathbb{R}\}$ is the greatest integer function, then the correct statement is :

- (A) f is continuous but not differentiable at $x = 2$.
- (B) f is neither continuous nor differentiable at $x = 2$.
- (C) f is continuous as well as differentiable at $x = 2$.
- (D) f is not continuous but differentiable at $x = 2$.

11. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to :

- (A) $2(\sin x + x \cos \theta) + C$
- (B) $2(\sin x - x \cos \theta) + C$
- (C) $2(\sin x + \sin \theta) + C$
- (D) $2(\sin x - x \sin \theta) + C$

12. $\int_0^1 \frac{2x}{5x^2 + 1} dx$ is equal to :

- (A) $\frac{1}{5} \log 6$
- (B) $\frac{1}{5} \log 5$
- (C) $\frac{1}{2} \log 6$
- (D) $\frac{1}{2} \log 5$

13. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

- (A) $(1, -10)$
- (B) $(1, 10)$
- (C) $(10, 1)$
- (D) $(-10, 1)$

14. The integrating factor of the differential equation

$$\frac{dx}{dy} = \frac{-(1 + \sin x)}{x + y \cos x} \text{ is :}$$

- (A) $\log \cos x$
- (B) $1 + \sin x$
- (C) $e^{(1 + \sin x)}$
- (D) $e^{\log \cos x}$

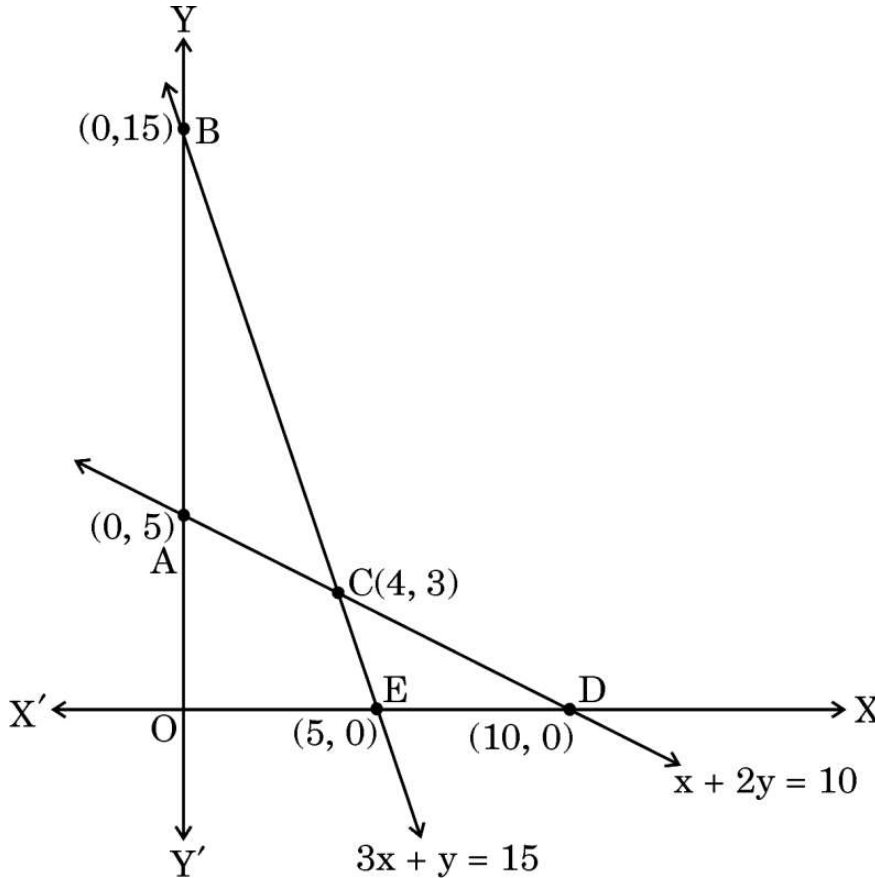


15. For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints :

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

$$x, y \geq 0$$



The correct feasible region is :

- (A) ABC (B) AOEC
(C) CED (D) Open unbounded region BCD
16. The sum of the order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = \frac{d^2y}{dx^2} \text{ is :}$$

- (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

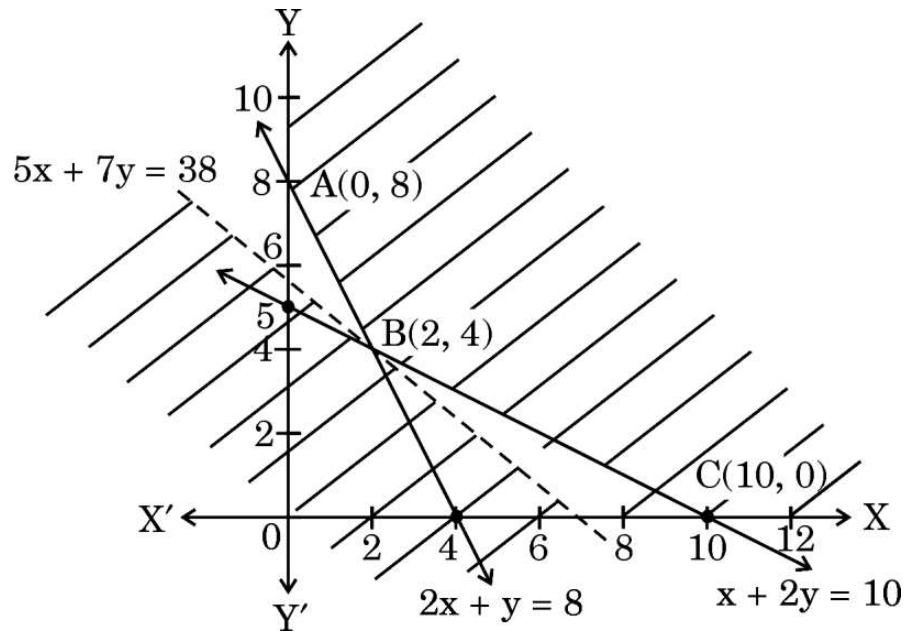
17. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :
- (A) 48 and 16 (B) 3 and 1
(C) 24 and 8 (D) 6 and 2



18. Let $\vec{a}$ be a position vector whose tip is the point $(2, -3)$. If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are $(-4, 5)$, then the coordinates of B are :
- (A) $(-2, -2)$ (B) $(2, -2)$ (C) $(-2, 2)$ (D) $(2, 2)$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.
19. Assertion (A) : The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



$$\text{Min } Z = 50x + 70y$$

subject to constraints

$$2x + y \geq 8, \quad x + 2y \geq 10, \quad x, y \geq 0$$

$$Z = 50x + 70y \text{ has a minimum value} = 380 \text{ at } B(2, 4).$$

Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.



20. Assertion (A) : Let $A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$. If $f : A \rightarrow A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm \sqrt{-1} \notin A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $\sin^{-1}(x^2 - 3)$.
22. Let the volume of a metallic hollow sphere be constant. If the inner radius increases at the rate of 2 cm/s, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm respectively.
23. A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, -2) and B (6, 2, -3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

24. (a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x .

OR

- (b) If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

25. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

OR

- (b) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.



SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

- 26.** Find the interval/intervals in which the function $f(x) = \sin 3x - \cos 3x$, $0 < x < \frac{\pi}{2}$ is strictly increasing.

- 27.** (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

- (b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ , then prove that $\frac{1}{2} |\vec{a} - \vec{b}| = \sin \frac{\theta}{2}$.

- 28.** Solve the differential equation

$$x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x.$$

- 29.** (a) The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is 0.3. Find the probability that the student :

- (i) Buys both the colouring book and the box of colours.
- (ii) Buys a box of colours given that she buys the colouring book.

OR

- (b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :

- (i) The probability distribution of the number of oranges he draws.
- (ii) The expectation of the random variable (number of oranges).



30. (a) Find :

$$\int \frac{2x}{(x^2 + 3)(x^2 - 5)} dx$$

OR

(b) Evaluate :

$$\int_1^4 (|x-2| + |x-4|) dx$$

31. In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$

subject to constraints

$$x + 2y \leq 120$$

$$x + y \geq 60$$

$$x - 2y \geq 0$$

$$x, y \geq 0$$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. In a rough sketch, mark the region bounded by $y = 1 + |x + 1|$, $x = -2$, $x = 2$ and $y = 0$. Using integration, find the area of the marked region.

33. Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.



- 34.** (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

- (b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

- 35.** (a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

- (b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (-1, -1, 2).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

- 36.** Let A be the set of 30 students of class XII in a school. Let $f: A \rightarrow N$, N is a set of natural numbers such that function $f(x)$ = Roll Number of student x. On the basis of the given information, answer the following :

- (i) Is f a bijective function ? 1
- (ii) Give reasons to support your answer to (i). 1



- (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where
 $R = \{(x, y) : x, y \text{ are Roll Numbers of students such that } y = 3x\}$.
List the elements of R . Is the relation R reflexive, symmetric and transitive ? Justify your answer. 2

OR

- (iii) (b) Let R be a relation defined by
 $R = \{(x, y) : x, y \text{ are Roll Numbers of students such that } y = x^3\}$.
List the elements of R . Is R a function ? Justify your answer. 2

Case Study – 2

37. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.



Radish



Cabbage



Brinjal

Based upon the above information, answer the following questions :

- (i) Calculate the probability of a randomly chosen seed to germinate. 2
- (ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? 2



Case Study – 3

38. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

- (i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. 1
- (ii) Find $\frac{dS}{dx}$. 1
- (iii) (a) Find a relation between x and y such that the surface area (S) is minimum. 2

OR

- (iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. 2